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LETTER TO THE EDITOR

Integrable model of interacting XX and Fateev–Zamolodchikov chains

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Abstract. We consider the exact solution of a model of correlated particles, which is presented as a system of interacting XX and Fateev–Zamolodchikov chains. This model can also be considered as a generalization of the multiband anisotropic t–J model in the case where we restrict the site occupations to at most two electrons. The exact solution is obtained for the eigenvalues and eigenvectors using the Bethe ansatz method.

It is well known that in the limit of a strong Coulomb repulsion, U, the traditional Hubbard model reduces to the so-called t–J model [1]. The strong on-site correlations limit the site occupations to at most one electron. States with double occupation on a given site are energetically unfavourable and can be projected out from the Hilbert space. In one dimension this model is integrable at the supersymmetrical point [2–5]. The exact solution can also be obtained for the extension of the supersymmetric spin-$\frac{1}{2}$ t–J model (i.e. $N = 2$) to the case of an arbitrary number N of bands, having a $SU(N)$ symmetry [6, 7], as well as for their anisotropic version [8, 9]. In some sense all these models can be considered as a system of interacting XX and XXZ chains with size occupations limited to at most one electron per site [2].

In this letter we consider a generalization of the multiband t–J model for the case where the on-site correlations are enough strong to limit the site occupations to at most two electrons. The exact integrable model we present can be considered as a system of interacting XX and spin-1 Fateev–Zamolodchikov chains [10, 11].

In order to present our model let us initially consider a general model with N distinct types of bands ($\alpha = 1, \ldots, N$) split into two disjoint groups $\mathbb{N}_1$ and $\mathbb{N}_2$. Taking into account the constraint of maximum double occupancy in a given site we can have at most one electron in bands $\alpha \in \mathbb{N}_1$ and at most two electrons in bands $\alpha \in \mathbb{N}_2$. The states of a given site are denoted by $|\alpha, \beta\rangle$ ($\alpha \leqslant \beta, \alpha, \beta = 0, 1, \ldots, N$), where in the case $\alpha, \beta = 1, 2, \ldots N$ these are the bands where the electrons are located while $\alpha = 0$ or $\beta = 0$ denotes the absence of electrons. As a conequence of this notation the state $|\alpha, \alpha\rangle$ is forbidden if $\alpha \in \mathbb{N}_1$ and it is allowed if $\alpha \in \mathbb{N}_2$ or $\alpha = 0$. The most general Hamiltonian with nearest-neighbour interactions, in a lattice with

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L sites and periodic boundary condition, that conserves separately the electrons in each band can be written as

$$\begin{aligned} H &= -\sum_{j=1}^{L} H_{j,j+1} \\ H_{j,j+1} &= \sum_{[\alpha],[\alpha']=0}^{N} W^{\alpha_1,\alpha_2,\alpha_3,\alpha_4}_{\alpha'_1,\alpha'_2,\alpha'_3,\alpha'_4} E^{\alpha_1,\alpha_2|\alpha'_1,\alpha'_2}_{j} E^{\alpha_3,\alpha_4|\alpha'_3,\alpha'_4}_{j+1} \end{aligned} \tag{1}$$

where $[\alpha] \equiv [\alpha'_1, \alpha'_2, \alpha'_3, \alpha'_4]$ is a permutation of $[\alpha_1, \alpha_2, \alpha_3, \alpha_4]$ which keeps the order

$$\alpha_1 \leqslant \alpha_2 \quad \alpha'_1 \leqslant \alpha'_2 \quad \alpha_3 \leqslant \alpha_4 \quad \alpha'_3 \leqslant \alpha'_4 \qquad \alpha_i = 0, 1, 2, \ldots, N.$$

The matrix $E^{\alpha,\beta|\alpha',\beta'}$ has all the elements zero except the element in the α, β line and the α', β' column, with unit value, i.e.

$$E^{\alpha,\beta|\alpha',\beta'} = |\alpha\beta\rangle\langle\alpha'\beta'|. \tag{2}$$

We consider Hermitean Hamiltonians,

$$W^{\alpha,\beta,\gamma,\delta}_{\alpha',\beta',\gamma',\delta'} = W^{\alpha'\beta'\gamma'\delta'}_{\alpha\beta\gamma\delta} \tag{3}$$

in the absence of external fields,

$$W^{0,\alpha,0,0}_{0,\alpha,0,0} = W^{0,0,0,\alpha}_{0,0,0,\alpha} = 0 \qquad 0 \leqslant \alpha \leqslant N \tag{4}$$

and satisfying the chirality property

$$W^{\alpha,\beta,\gamma,\delta}_{\alpha',\beta',\gamma',\delta'}(\eta) = W^{\gamma,\delta,\alpha,\beta}_{\gamma',\delta',\alpha',\beta'}(-\eta) \tag{5}$$

where η is the asymmetry parameter. We choose the energy scale such that all single-particle hopping couplings have a unit value,

$$W^{0,0,0,\alpha}_{0,\alpha,0,0} = t = 1 \qquad 0 < \alpha \leqslant N. \tag{6}$$

Let us initially consider the case of a single particle on the otherwise empty chain. As a consequence of translational invariance of the Hamiltonian (1) the eigenfunctions are the plane waves of wavenumber k,

$$\begin{aligned} \Psi_1 &= \sum_{x_1} f(x_1, \alpha_1) E^{0,\alpha_1|0,0}_{x_1} |0, \ldots, 0\rangle \\ f(x_1, \alpha_1) &= \mathrm{e}^{\mathrm{i}kx} \qquad k = \frac{2\pi}{L} l \qquad (l = 0, 1, \ldots, L-1) \end{aligned} \tag{7}$$

with energy

$$E = -2\cos k. \tag{8}$$

In (7) $|0, \ldots, 0\rangle$ is the reference state with no particles.

The eigenfunctions of the Hamiltonian (1) in the general case, where we have n particles,

$$\Psi_n = \sum_{\{Q\}} \sum_{\{x\}} f(x_1, \alpha_1; x_2, \alpha_2; \ldots; x_n, \alpha_n) E^{0,\alpha_1|0,0}_{x_1} E^{0,\alpha_2|0,0}_{x_2} \ldots E^{0,\alpha_n|0,0}_{x_n} |0. \ldots, 0\rangle \tag{9}$$

are calculated by using a generalized nested Bethe ansatz. In (9) the first summation is over all the permutations $Q = [Q_1, Q_2, \ldots, Q_n]$ of the integers $1, 2, \ldots, N$, and for a given permutation $[Q]$ the second sum is restricted to the set $1 \leqslant x_{Q_1} \leqslant x_{Q_2} \leqslant \cdots \leqslant x_{Q_n}$. We seek the amplitudes f in each of these regions in the form of a superposition of plane waves with wavenumbers k_j, $j = 1, \ldots, n$. If we have only single occupation ($x_{Q_i} \neq x_{Q_{i+1}}, i = 1, 2, \ldots, n-1$) we write the ansatz

$$f(x_1, \alpha_{Q_1}; x_2, \alpha_{Q_2}; \ldots; x_n, \alpha_{Q_n}) = \sum_{P} A^{\alpha_{Q_1} \ldots \alpha_{Q_n}}_{P_1 \ldots P_n} \prod_{j=1}^{n} \exp(\mathrm{i}k_{P_j} x_{Q_j}) \tag{10}$$

where the sum is over all permutations $P = [P_1, \ldots, P_n]$ of the integers $1, 2, \ldots, n$. In the case where we have a pair at the position $x_{Q_l} = x_{Q_{l+1}}$, the ansatz is modified to

$$f(x_1, \alpha_{Q_1}; x_2, \alpha_{Q_2}; \ldots; x_n, \alpha_{Q_n}) = \sum_P A^{\alpha_{Q_1} \ldots \overline{\alpha_{Q_l} \alpha_{Q_{l+1}}} \ldots \alpha_{Q_n}}_{P_1 \ldots P_l P_{l+1} \ldots P_n} \prod_{j=1}^{n} \exp(\mathrm{i} k_{P_j} x_{Q_j}) \tag{11}$$

where the bar at the lth and $(l+1)$th positions of the superscript indicates the pair location. The general case with many isolated particles and pairs follows from (10) and (11). The ansatz (10), (11) corresponding to the configurations where $|x_{Q_{i+1}} - x_{Q_i}| \rangle 1$ satisfies $H|\Psi\rangle = E|\Psi\rangle$, providing the energy and momentum are given by

$$E = -2 \sum_{j=1}^{n} \cos k_j \qquad P = \sum_{j=1}^{n} k_j. \tag{12}$$

In order to obtain the two-particle scattering matrix we consider the general amplitudes (10) or (11) in the case where there are only two particles on two neighbouring sites. In this case our problem is exactly equivalent to that of a model with only two species (bands). The coefficients $A^{\alpha_{Q_1} \ldots \alpha_{Q_n}}_{P_1 \ldots P_n}$ arising from the different permutation Q are connected with each other by the elements of the two-particle S-matrix:

$$A^{\ldots \alpha \beta \ldots}_{\ldots P_1 P_2 \ldots} = \sum_{\delta, \gamma = 1}^{N} S^{\gamma \delta}_{\alpha \beta}(k_{P_1}, k_{P_2}) A^{\ldots \delta \gamma \ldots}_{\ldots P_2 P_1 \ldots}. \tag{13}$$

As a necessary condition to ensure integrability, the two-particle scattering matrix has to satisfy the Yang–Baxter equation [12, 13]. There are two integrable models, described in terms of two species of particles, whose S-matrix has a factorizable form. The first of these models is the standard Hubbard model [13], while the second one is the correlated hopping model [14–16]. We could try to use as the fundamental building block of a general S-matrix of our model (1) the two-particle scattering matrix of these models. Here we consider the second model and choose the S-matrix as in the correlated hopping model [16]†. In this case we have the following restrictions on the parameters of the Hamiltonian (1):

$$\begin{aligned}
&W^{0,\alpha,0,\beta}_{0,\alpha,0,\beta} = 0, \qquad 0 < \alpha < \beta \\
&W^{\alpha,\beta,0,0}_{\alpha,\beta,0,0} = \epsilon_0 \qquad 0 < \alpha \leqslant \beta \\
&W^{\alpha,\beta,0,0}_{0,\alpha,0,\beta} = W^{0,0,\alpha,\beta}_{0,\alpha,0,\beta} = \sqrt{1 + \mathrm{e}^{2\eta}} \qquad 0 < \alpha < \beta \\
&W^{\alpha,\alpha,0,0}_{0,\alpha,0,\alpha} = 2\epsilon_1 \cosh \eta \qquad \alpha \in \mathbb{N}_2 \\
&W^{0,0,\alpha,\beta}_{\alpha,\beta,0,0} = \epsilon_0 \qquad 0 < \alpha \leqslant \beta
\end{aligned} \tag{14}$$

where $\epsilon_0 = \pm 1$ and $\epsilon_1 = \pm 1$. The non-vanishing elements of the S-matrix are

$$\begin{aligned}
S^{\alpha\beta}_{\alpha\beta}(k_1, k_2) &= -\frac{\sin(\lambda_1 - \lambda_2)}{\sin(\lambda_1 - \lambda_2 - \mathrm{i}\eta)} \\
S^{\alpha\beta}_{\beta\alpha}(k_1, k_2) &= \frac{-\mathrm{i} \sinh \eta}{\sin(\lambda_1 - \lambda_2 - \mathrm{i}\eta)} \exp[\mathrm{i} \mathrm{sign}(\alpha - \beta)(\lambda_1 - \lambda_2)] \\
S^{\alpha\alpha}_{\alpha\alpha}(k_1, k_2) &= -\frac{\sin[-\varepsilon_i(\lambda_1 - \lambda_2) - \mathrm{i}\eta]}{\sin(\lambda_1 - \lambda_2 - \mathrm{i}\eta)}
\end{aligned} \tag{15}$$

† Unfortunately previous attempts to solve this problem by using the Hubbard S-matrix was unsuccessful [17] since the Bethe ansatz does not work in the sector where we have three or more particles. The interactions necessary to prevent more than double ocupancy per site spoils the exact integrability. This is not the case if we use as the fundamental building block the S-matrix of the correlated hopping model.

where

$$e^{ik_j} = -\epsilon_0 \frac{\sin(\lambda_j + i\eta)}{\sin(\lambda_j - i\eta)}. \tag{16}$$

To complete the proof of the Bethe ansatz (10), (11) we must consider the eigenvalue equations in the case where there are three and four particles on the two neighbouring sites j and $j+1$. This gives us a complicated system of equations for those parameters of the Hamiltonian (1) involving three and four particles. We have treated this system analytically and checked our results numerically. We found the following solution for the diagonal elements of W:

$$\begin{aligned}
&W^{\alpha,\beta,0,\gamma}_{\alpha,\beta,0,\gamma} = \epsilon_0(1-e^{2\eta}) \qquad 0<\alpha\leqslant\beta<\gamma \\
&W^{\alpha,\beta,0,\gamma}_{\alpha,\beta,0,\gamma} = \epsilon_0(1-e^{-2\eta}) \qquad 0<\gamma<\alpha\leqslant\beta \\
&W^{\alpha,\beta,0,\gamma}_{\alpha,\beta,0,\gamma} = 0 \qquad 0<\alpha<\gamma<\beta \\
&W^{\alpha,\beta,0,\beta}_{\alpha,\beta,0,\beta} = \begin{cases} \epsilon_0 & \beta\in\mathbb{N}_1 \\ -\epsilon_0 e^{2\eta} & \beta\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta \\
&W^{\alpha,\beta,0,\alpha}_{\alpha,\beta,0,\alpha} = \begin{cases} \epsilon_0 & \alpha\in\mathbb{N}_1 \\ -\epsilon_0 e^{-2\eta} & \alpha\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta \\
&W^{\alpha,\beta,\gamma,\delta}_{\alpha,\beta,\gamma,\delta} = \epsilon_0(1-2e^{2\eta}) \qquad 0<\alpha\leqslant\beta<\gamma\leqslant\delta \\
&W^{\alpha,\beta,\gamma,\delta}_{\alpha,\beta,\gamma,\delta} = -\epsilon_0 e^{2\eta} \qquad 0<\alpha<\gamma<\beta<\delta \\
&W^{\alpha,\beta,\gamma,\delta}_{\alpha,\beta,\gamma,\delta} = \epsilon_0(1-2\cosh 2\eta) \qquad 0<\alpha<\gamma\leqslant\delta<\beta \\
&W^{\alpha,\beta,\beta,\gamma}_{\alpha,\beta,\beta,\gamma} = \begin{cases} \epsilon_0(1-e^{2\eta}) & \beta\in\mathbb{N}_1 \\ -2\epsilon_0 e^{2\eta} & \beta\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta<\gamma \\
&W^{\alpha,\gamma,\beta,\gamma}_{\alpha,\gamma,\beta,\gamma} = \begin{cases} \epsilon_0(1-e^{2\eta}) & \gamma\in\mathbb{N}_1 \\ -2\epsilon_0\cosh 2\eta & \gamma\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta<\gamma \\
&W^{\alpha,\beta,\alpha,\gamma}_{\alpha,\beta,\alpha,\gamma} = \begin{cases} \epsilon_0(1-e^{2\eta}) & \alpha\in\mathbb{N}_1 \\ -2\epsilon_0\cosh 2\eta, & \alpha\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta<\gamma \\
&W^{\alpha,\beta,\alpha,\beta}_{\alpha,\beta,\alpha,\beta} = \begin{cases} 2\epsilon_0 & \alpha,\beta\in\mathbb{N}_1 \\ -2\epsilon_0\cosh 2\eta & \alpha \text{ or } \beta\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta \\
&W^{\alpha,\alpha,\alpha,\beta}_{\alpha,\alpha,\alpha,\beta} = -\epsilon_0(2e^{2\eta}+e^{-2\eta}) \qquad \alpha\in\mathbb{N}_2 \qquad 0<\alpha<\beta \\
&W^{\beta,\beta,\alpha,\beta}_{\beta,\beta,\alpha,\beta} = -\epsilon_0(e^{2\eta}+2e^{-2\eta}) \qquad \beta\in\mathbb{N}_2 \qquad 0<\alpha<\beta \\
&W^{\alpha,\alpha,0,\alpha}_{\alpha,\alpha,0,\alpha} = \tfrac{1}{2}W^{\alpha,\alpha,\alpha,\alpha}_{\alpha,\alpha,\alpha,\alpha} = -2\epsilon_0\cosh 2\eta \qquad \alpha\in\mathbb{N}_2.
\end{aligned} \tag{17}$$

For the non-diagonal elements of W we have

$$\begin{aligned}
&W^{0,\alpha,\beta,\gamma}_{\alpha,\beta,0,\gamma} = 1 \qquad 0<\alpha<\beta<\gamma \\
&W^{0,\beta,\alpha,\gamma}_{\alpha,\beta,0,\gamma} = W^{\alpha,\gamma,0,\beta}_{0,\alpha,\beta,\gamma} = e^{-\eta} \qquad 0<\alpha<\beta<\gamma \\
&W^{0,\beta,\alpha,\beta}_{\alpha,\beta,0,\beta} = \begin{cases} 2\cosh\eta & \beta\in\mathbb{N}_1 \\ 0 & \beta\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta \\
&W^{0,\alpha,\alpha,\beta}_{\alpha,\beta,0,\alpha} = \begin{cases} 2\cosh\eta & \alpha\in\mathbb{N}_1 \\ 0 & \alpha\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha<\beta \\
&W^{0,\alpha,\alpha,\beta}_{\alpha,\alpha,0,\beta} = W^{\alpha,\beta,0,\beta}_{0,\alpha,\beta,\beta} = \epsilon_1\sqrt{1+e^{-2\eta}} \qquad 0<\alpha<\beta
\end{aligned}$$

$$
\begin{aligned}
&W^{\alpha,\gamma,0,\beta}_{\alpha,\beta,0,\gamma} = W^{0,\beta,\alpha,\gamma}_{0,\alpha,\beta,\gamma} = -\epsilon_0 \mathrm{e}^{\eta} \qquad 0<\alpha<\beta<\gamma \\
&W^{\beta,\gamma,0,\alpha}_{\alpha,\beta,0,\gamma} = -\epsilon_0 \qquad 0<\alpha<\beta<\gamma \\
&W^{\alpha,\beta,0,\alpha}_{\alpha,\alpha,0,\beta} = W^{0,\beta,\alpha,\beta}_{0,\alpha,\beta,\beta} = -\epsilon_0\epsilon_1\sqrt{1+\mathrm{e}^{2\eta}} \qquad 0<\alpha<\beta \qquad (18) \\
&W^{0,\gamma,\alpha,\beta}_{\alpha,\beta,0,\gamma} = -1 \qquad 0<\alpha\leqslant\beta \qquad \gamma\neq\alpha,\beta \\
&W^{\alpha,\delta,\beta,\gamma}_{\alpha,\beta,\gamma,\delta} = W^{\beta\gamma\alpha\delta}_{\alpha\beta\gamma\delta} = -\epsilon_0 \qquad 0<\alpha<\beta<\gamma<\delta \\
&W^{\alpha,\gamma,\beta,\delta}_{\alpha,\beta,\gamma,\delta} = W^{\gamma,\delta,\alpha,\beta}_{\alpha,\gamma,\beta,\delta} = W^{\alpha,\delta,\beta,\gamma}_{\beta,\delta,\alpha,\gamma} W^{\beta,\gamma,\alpha,\delta}_{\beta,\delta,\alpha,\gamma} = -\epsilon_0\mathrm{e}^{\eta} \qquad 0<\alpha<\beta<\gamma<\delta \\
&W^{\alpha,\beta,\alpha,\gamma}_{\alpha,\alpha,\beta,\gamma} = W^{\alpha,\gamma,\beta,\gamma}_{\alpha,\beta,\gamma,\gamma} = W^{\alpha,\beta,\alpha,\gamma}_{\beta,\gamma,\alpha,\alpha} = W^{\alpha,\gamma,\beta,\gamma}_{\gamma,\gamma,\alpha,\beta} = W^{\beta,\gamma,\alpha,\beta}_{\beta,\beta,\alpha,\gamma} = W^{\beta,\gamma,\alpha,\beta}_{\alpha,\gamma,\beta,\beta} = W^{\beta,\gamma,\alpha,\beta}_{\alpha,\gamma,\beta,\beta} W^{\beta,\gamma,\alpha,\beta}_{\beta,\beta,\alpha,\gamma} \\
&\qquad = -\epsilon_0\epsilon_1\sqrt{1+\mathrm{e}^{2\eta}} \qquad 0<\alpha<\beta<\gamma \\
&W^{\beta,\gamma,\alpha,\beta}_{\alpha,\beta,\beta,\gamma} = W^{\gamma,\beta,\alpha,\beta}_{\alpha,\beta,\gamma,\beta} = W^{\beta,\gamma,\beta,\alpha}_{\beta,\alpha,\beta,\gamma} = \begin{cases} -2\epsilon_0\cosh\eta & \beta\in\mathbb{N}_1 \\ 0 & \beta\in\mathbb{N}_2 \end{cases} \qquad 0<\alpha\neq\beta\neq\gamma \\
&W^{\alpha,\alpha,\alpha,\beta}_{\alpha,\beta,\alpha,\alpha} = W^{\beta,\beta,\alpha,\beta}_{\alpha,\beta,\beta,\beta} = -\epsilon_0 \qquad 0<\alpha<\beta \\
&W^{\alpha,\alpha,\beta,\beta}_{\alpha,\beta,\alpha,\beta} = -2\epsilon_0\cosh\eta \qquad 0<\alpha<\beta \\
&W^{\gamma,\delta,\alpha,\beta}_{\alpha,\beta,\gamma,\delta} = \epsilon_0 \qquad \alpha,\beta\neq\gamma,\delta.
\end{aligned}
$$

The periodic boundary condition for the system on the finite lattice, with size L, gives us the Bethe ansatz equations. In order to obtain these equations we must diagonalize the transfer matrix of a related inhomogeneous vertex model with non-intersecting strings [18]. The Bethe-ansatz equations are written in terms of the momenta of the electrons k_j and additional rapidities $\Lambda^{(i)}_\alpha$:

$$
\begin{aligned}
&\left[\frac{\sin(\lambda_j+\mathrm{i}\eta)}{\sin(\lambda_j-\mathrm{i}\eta)}\right]^L = \varepsilon_1^{n-m_1-1}\prod_{j'=1}^{n}\frac{\sin(\lambda_j-\lambda_{j'}+\mathrm{i}\varepsilon_1\eta)}{\sin(\lambda_j-\lambda_{j'}-\mathrm{i}\eta)}\prod_{\alpha=1}^{m_1}\frac{\sin(\lambda_j-\Lambda^{(1)}_\alpha-\frac{\mathrm{i}}{2}\varepsilon_1\eta)}{\sin(\lambda_j-\Lambda^{(1)}_\alpha+\frac{\mathrm{i}}{2}\varepsilon_1\eta)} \\
&\prod_{\alpha'=1}^{m_\sigma}\frac{\sin(\Lambda^{(\sigma)}_\alpha-\Lambda^{(\sigma)}_{\alpha'}+\mathrm{i}\varepsilon_{\sigma+1}\eta)}{\sin(\Lambda^{(\sigma)}_\alpha-\Lambda^{(\sigma)}_{\alpha'}-\mathrm{i}\varepsilon_\sigma\eta)} = \varepsilon_\sigma^{m_{\sigma-1}+m_\sigma-1}\varepsilon_{\sigma+1}^{m_{\sigma+1}+m_\sigma-1}\prod_{\alpha'=1}^{m_{\sigma-1}}\frac{\sin(\Lambda^{(\sigma)}_\alpha-\Lambda^{(\sigma-1)}_\alpha+\frac{\mathrm{i}}{2}\varepsilon_\sigma\eta)}{\sin(\Lambda^{(\sigma)}_\alpha-\Lambda^{(\sigma-1)}_\alpha-\frac{\mathrm{i}}{2}\varepsilon_\sigma\eta)} \qquad (19) \\
&\qquad\qquad \times\prod_{\alpha'=1}^{m_{\sigma+1}}\frac{\sin(\Lambda^{(\sigma)}_\alpha-\Lambda^{(\sigma+1)}_\alpha+\frac{\mathrm{i}}{2}\varepsilon_{\sigma+1}\eta)}{\sin(\Lambda^{(\sigma)}_\alpha-\Lambda^{(\sigma+1)}_\alpha-\frac{\mathrm{i}}{2}\varepsilon_{\sigma+1}\eta)} \\
&\sigma=1,2,\ldots,N-1 \quad m_0=n \quad m_N=0 \quad \Lambda^{(0)}_\alpha=\lambda_\alpha
\end{aligned}
$$

where $n_j = m_{j-1} - m_j$ is the number of particles of species j, $\varepsilon_\sigma = +1$ if $\sigma\in\mathbb{N}_1$ and $\varepsilon_\sigma = -1$ for $\sigma\in\mathbb{N}_2$. The energy of the system is given in terms of the Bethe ansatz roots λ_j

$$
E = -2\sum_{j=1}^{n}\cos k_j = 2\epsilon_0\sum_{j=1}^{n}\left[\cosh 2\eta - \frac{\sinh^2 2\eta}{\cosh 2\eta - \cos 2\lambda_j}\right]. \qquad (20)
$$

It is interesting to remark that the particular isotropic cases $\gamma = 0$, of (19), (20), where $N = 2$ and $\varepsilon_1 = -\varepsilon_2 = 1$, as well as for arbitrary N, but $\varepsilon_i = 1 (i = 1,\ldots,N)$ have been obtained in [19, 20], respectively.

In conclusion, we have studied a new integrable model which can be presented as a system of interacting XX and Fateev–Zamolodchikov chains. The Bethe ansatz equations are derived by means of the coordinate Bethe ansatz approach. A desirable continuation of the present work is the investigation of the thermodynamic equilibrium properties of the model based on the Bethe ansatz solutions (19), (20).

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